Entanglement evolution of a two-qubit system with decay beyond rotating-wave approximation

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Two noninteracting atoms, initially entangled in Bell states, are coupled to a one-mode cavity. Based on the reduced non-perturbative quantum master equation, the entanglement evolution of the two atoms with decay is investigated beyond rotating-wave approximation. It is shown that the counter-rotating wave terms have great influence on the disentanglement behavior. The phenomenon of entanglement sudden death and entanglement sudden birth will occur.

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I. INTRODUCTION

Entanglement plays a central role in the field of quantum information science where it is considered as a valuable resource for some non-classical tasks, such as: quantum computation[1, 2, 3], quantum teleportation [4], superdense coding[5], and quantum cryptography [6, 7]. Applications of interest have triggered research on the dynamical behavior of entanglement in order to control quantum disentanglement[8, 9, 10].

Recently, the dynamics of entanglement in bi-partite systems has been under extensive research [11, 12, 13, 14, 15, 16, 17, 18, 19, 20] since the work of Yu and Eberly[21], in which the entanglement may terminate abruptly in a finite time while coherence is lost asymptotically. This effect is termed entanglement sudden death(ESD). Subsequently, C.E.Lo'pez *etc.* put forward a new term "entanglement sudden birth"(ESB)[22] and try to present an explanatory study of multipartite entanglement evolution.

In the previous studies, the rotating-wave approximation(RWA), which neglecting counter rotating terms corresponding to the emission and absorption of virtual photon without energy conservation, is widely used. Generally, the coupling ratio of the atom-field is of the order $10^{-7} \sim 10^{-6}$ in atom-field cavity systems and the RWA is justified. Recently, D.Meiser *etc.* have investigated the cavity systems with superstrong coupling[23]. It can be seen that the effect of counter-rotating terms should be considered in the strong coupling regime, such as solid state systems.

Different from the previous works, we study the disentanglement between a pair of qubits with decay which interacting with a cavity beyond the RWA. Considering the real processing, the atomic decay is unavoidable, which can be caused by two different physical mechanisms. The first is the transition without light radiation. In this process the energy is emitted by thermal energy or the other forms. The second is the radiation transition. There will emit photons with the atomic transition. In our model the second is the main physical mechanism because the collision probability between the two atoms is very small. Thus the discussion upon the effect of atomic decay caused by spontaneous emission to the entanglement evolution is necessary.

The paper is organized as follows. In section II, the two-qubit model with decay is presented and the reduced non-perturbative quantum master equation of atoms is derived. In section III, the entanglement evolution of two initially entangled atoms is investigated by using Wootters' concurrence[24]. In the last section, the conclusions are given.

II. MODEL

Consider two noninteracting atoms A and B, which are interacting with a single-mode cavity resonantly. For simplicity, we assume that the two atoms have the same parameters. It can be described by the following Hamiltonian:

$$H = H_a + H_f + H_{af}, \tag{1}$$

where

$$H_a = \frac{\omega_0}{2}\Sigma_{i=1}^2 \sigma_{z_i}, \tag{2}$$

$$H_f = \omega a^+ a, \tag{3}$$

$$H_{af} = \lambda\sigma(a^+ + a), \tag{4}$$

where $\sigma = \Sigma_{i=1}^2(\sigma_i^+ + \sigma_i^-)$, $\sigma_{z_i} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$ and $\sigma_i^- = |g_i\rangle\langle e_i|$ are the atomic operators with $|e_i\rangle$ and $|g_i\rangle$ being the excited and ground states of the ith atom; ω_0 is the atomic transition frequency between the ground state and the excited state; λ is the coupling constant between atom and cavity; a and $a^\dagger$ denote the annihilation and creation operators of the cavity field mode corresponding frequency ω.

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Taking the atomic spontaneous emission into consideration, the reduced non-perturbative quantum master equation of atoms could be obtained by path integrals[25],

$$\begin{aligned}\frac{\partial}{\partial t}\rho_a = & - i\mathcal{L}_a\rho_a - \int_0^t ds\langle\mathcal{L}_{af}e^{-i\mathcal{L}_0(t-s)}\mathcal{L}_{af}e^{-i\mathcal{L}_0(s-t)}\rangle_f\rho_a \\ & + \frac{\gamma}{2}\Sigma_{i=1}^2(2\sigma_i^-\rho_a\sigma_i^+ - \sigma_i^+\sigma_i^-\rho_a - \rho_a\sigma_i^+\sigma_i^-), \end{aligned} \quad (5)$$

where $\mathcal{L}_0$, $\mathcal{L}_a$ and $\mathcal{L}_{af}$ are Liouvillian operators defined as

$$\begin{aligned}\mathcal{L}_0\rho &= [H_a + H_f, \rho], \\ \mathcal{L}_a\rho &= [H_a, \rho], \\ \mathcal{L}_{af}\rho &= [H_{af}, \rho], \end{aligned} \quad (6)$$

and $< \cdots >_f$ stands for partial trace of cavity mode; γ is the atomic decay constant.

Assuming that the cavity field is initially in vacuum state, the non-perturbative reduced master equation of the atoms could be derived from Eq.(5),

$$\begin{aligned}\frac{\partial}{\partial t}\rho_a = & - i\frac{\omega_0}{2}[\sigma_{z_1} + \sigma_{z_2}, \rho] \\ & - \alpha\lambda^2\sigma[\sigma_1^+ + \sigma_2^+, \rho] - f\lambda^2\sigma[\sigma_1^- + \sigma_2^-, \rho] \\ & + \alpha^*\lambda^2[\sigma_1^- + \sigma_2^-, \rho]\sigma + f^*\lambda^2[\sigma_1^+ + \sigma_2^+, \rho]\sigma \\ & + \frac{\gamma}{2}\Sigma_{i=1}^2(2\sigma_i^-\rho_a\sigma_i^+ - \sigma_i^+\sigma_i^-\rho_a - \rho_a\sigma_i^+\sigma_i^-) \end{aligned} \quad (7)$$

where

$$\begin{aligned}\alpha &= \frac{1 - exp(-i\Delta t)}{i\Delta}, \\ f &= \frac{exp(i\delta t) - 1}{i\delta}. \end{aligned} \quad (8)$$

and $\Delta = \omega + \omega_0$, $\delta = \omega_0 - \omega$, $f^*$is conjugate of f.

III. DISENTANGLEMENT

A. Initial state and the entanglement measurement

In this section, we will use Wootters' concurrence to quantify the degree of entanglement[24]. For two qubits, the concurrence is calculated from the density matrix ρ for qubits A and B:

$$C(\rho) = max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (9)$$

where λ_i are the eigenvalues of the matrix

$$\varrho = \rho_{AB}(\sigma_y^A \otimes \sigma_y^B)\rho_{AB}^*(\sigma_y^A \otimes \sigma_y^B) \quad (10)$$

arranged in decreasing order. Here ρ_{AB}^* denotes the complex conjugation of ρ_{AB}, and $\sigma_y^{A(B)}$ is the standard Pauli matrix acting in the space of qubit A (or B). The concurrence varies from $C(\rho) = 0$ for an unentangled state to $C(\rho) = 1$ for a maximally entangled state.

Here we restrict our analysis to the initial entangled states

$$|\phi\rangle = \alpha|01\rangle + \beta|10\rangle, |\psi\rangle = \alpha|00\rangle + \beta|11\rangle \quad (11)$$

where α is real, $\beta = |\beta|e^{i\delta}$ and $\alpha^2 + |\beta|^2 = 1$. For these two entangled states, the initial atomic density matrix has an "X" structure[26] which is maintained during the evolution[27]. The reduced density matrix of the two atoms ρ_a, in the standard product basis $\mathcal{B} = \{|1\rangle = |ee\rangle, |2\rangle = |eg\rangle, |3\rangle = |ge\rangle, |4\rangle = |gg\rangle\}$, could be written as

$$\rho_a = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (12)$$

The concurrence of ρ_a could be obtained

$$\begin{aligned}C_\phi(t) &= max\{0, 2|\rho_{23}| - 2\sqrt{\rho_{11}\rho_{44}}\}, \\ C_\psi(t) &= max\{0, 2|\rho_{14}| - 2\sqrt{\rho_{22}\rho_{33}}\}. \end{aligned} \quad (13)$$

B. Numerical results and discussion

In order to study the counter rotating terms effect on the system, we investigate the entanglement evolution of two atoms by numerical calculation. For simplicity, the resonant case $\omega = \omega_0$ is considered.

First, we focus on the disentanglement of two qubits with the initial state of $|\phi\rangle$. The entanglement evolution of two decayed atoms is shown in Fig.1, Fig.2 and Fig.3 with different parameters. One could see that the change of C_ϕ against α^2 is symmetrical because of the symmetry of the initial state $|\phi\rangle$.

When $\omega = \lambda$, Fig.1 shows that the concurrence C_ϕ changes with the initial value α^2 and the time λt. It can be seen that the concurrence decreases to zero in a finite time and the entanglement undergoes the so-called ESD. The tendency is similar to that of the above case when $\omega = 3\lambda$, as shown in Fig.2.

As $\omega = 10\lambda$, Fig.3 reveals the time evolvement of concurrence. The tendency is different from that of the above cases. It is easy to note that the entanglement decays to zero in a finite time, then revives with small amplitude and disappears permanently at last. Particularly, when the two atoms are not entangled initially, namely $\alpha^2 = 0$ or $\alpha^2 = 1$, there is entanglement sudden birth and its amplitude is larger than that of the case $0 < \alpha^2 < 1$, then the entanglement decreases to zero eventually due to the atomic decay.

Alternatively, we focus on the disentanglement of two qubits with the initial state $|\psi\rangle$. The entanglement evolvement is investigated in Fig.4, Fig.5 and Fig.6. The figures are not symmetrical to α^2 because the initial state of the two atoms is asymmetrical.

When $\omega = \lambda$, Fig.4 shows that the concurrence C_ψ decays exponentially to zero in almost all cases except that

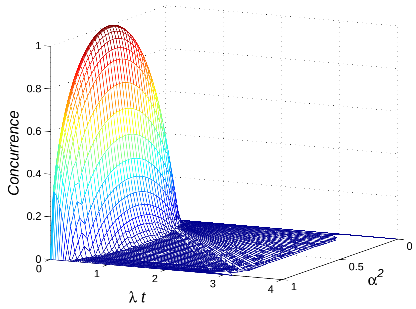

FIG. 1: The concurrence C_ϕ as functions of α^2 and the time λt with $\omega = \lambda$.

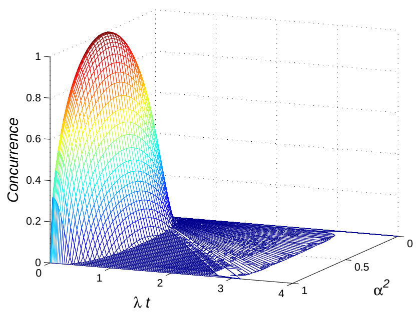

FIG. 2: The concurrence C_ϕ as functions of α^2 and the time $\lambda_A t$ with $\omega = 3\lambda$.

the value of α^2 is near 0. In the case of small α^2 , there exists small fluctuation before C_ψ vanishes permanently. Particularly, when $\alpha^2 = 0$, there exists small ESB and ESD. This effect is resulted from the coaction of rotating wave process and counter-rotating process on the whole system.

By contrast, as $\omega = 3\lambda$, Fig.5 reveals that the amplitude of fluctuation is higher than that of $\omega = \lambda$ in the case of small α^2. It is also found that the range existing the fluctuation is larger than that of the previous case. But there is no ESB in the case of $\alpha^2 = 0$.

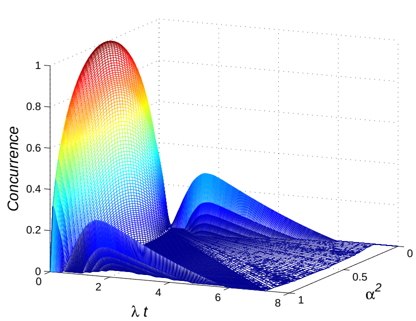

FIG. 3: The concurrence C_ϕ as functions of α^2 and the time $\lambda_A t$ with $\omega = 10\lambda$.

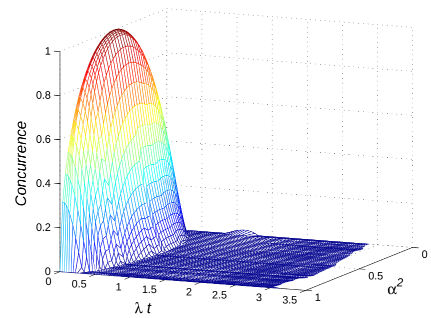

FIG. 4: The concurrence C_ψ as functions of α^2 and the time $6^{1/2}\lambda t$ with $\omega = \lambda$.

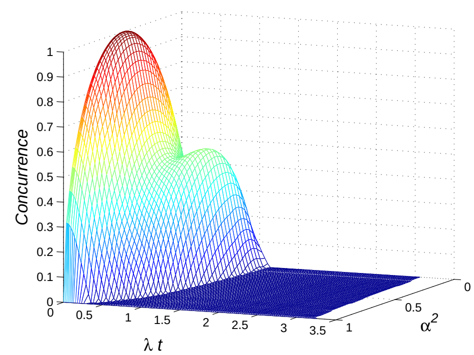

FIG. 5: The concurrence C_ψ as functions of α^2 and the time λt for $\omega = 3\lambda$.

When $\omega = 10\lambda$, the time evolution of the concurrence is plotted in Fig.6. The concurrence decreases monotonically and exponentially to zero when α^2 is near 1. The smaller the α^2 is, the more intense the fluctuation is. The concurrence disappears permanently at last because the atoms decay.

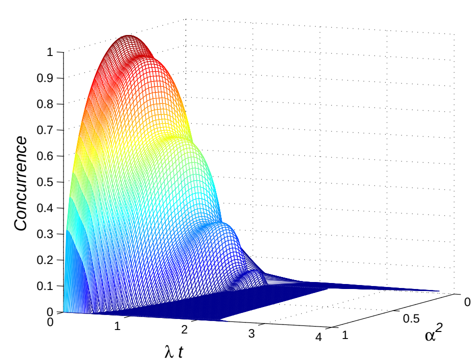

FIG. 6: The concurrence C_ψ as functions of α^2 and the time λt for $\omega = 10\lambda$.

IV. CONCLUSIONS

In summary, we have investigated the disentanglement of two noninteracting atoms with decay coupling to a one-mode cavity resonantly beyond RWA. It is shown that the entanglement evolution is dependent on the ratio of the atom-field coupling divided by the atomic transition frequency and the phenomena of entanglement evolution are rich. The physical mechanism behind the phenomena is the process of emission and absorption of virtual photon and the atomic decay.

The study of entanglement evolution beyond RWA is a significant problem because of its importance to the field of strong coupling. It will help ones to deal with the practical case of solid state system. The consideration of atomic decay is also of practical significance.

Acknowledgments

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